The Asian International School
Unit Backward Design
Mathematics, Elementary, 2018-2019
Chapter 1, Unit 1: Proportional Relations

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- recognize and represent proportional relationships between quantities
- decide whether two quantities are in a proportional relationship

Understanding(s):
Students will understand that…

- a ratio is an ordered pair of numbers which are not both zero.
- a ratio is denoted by A : B to indicate the order of the numbers
- two ratios A:B and C:D are equivalent ratios if there is a nonzero number c such that C = cA and D = cB
- the numerical part of the rate is called the **unit rate** and is simply the value of the ratio

Essential Question(s):

- When do we use irrational numbers?
- Why are irrational numbers important?

Knowledge:
Students will know …

- the set of rational numbers and the set of irrational numbers are disjoint set,
- irrational numbers cannot be expressed as a ratio between two numbers and cannot be written as a simple fraction.

Skills:
Student will be able to:

- compute unit rates
- simplify square roots
- estimate square roots
- identify positions of irrational number in the real number line

Stage 2 - Assessment Evidence

Performance Task(s):

- Using the concept of irrational numbers, you are going to design a board game for the class. The game must include the application of irrational numbers in real life. Your grade will be based on how much information on irrational numbers will be used to play the game.

Other Evidence:

- Homework
- Mini-Tests

Stage 3 – Learning Plan

Learning Activities:

Session 1: An experience in Relationships as Measuring Rate

- To start the lesson, have the students do an activity on passing their papers. This activity can be used to establish routine in passing papers during exams or paper activities.
- Determine how papers will be passed out in class depending upon seating arrangement. For this task, it is best to divide the original stack so that one student (in each row or group) has a portion of the original stack. Based upon this determination, explain the system to students. A brief demonstration may help to provide a visual.
- For example: If the room is arranged in rows, pass *across* the rows. Have students start on command and perhaps require that only the current paper-passing student may be out of his or her seat. If the room is arranged in groups or at tables, have the students pass papers to their left, on command, until everyone has a paper. *Note: This procedure is highly customizable for use in any classroom structure.*
- Begin the task by handing a stack of papers to a starting person. Secretly start a stopwatch as the start command is given. Once every student has a paper, report the paper-passing time out loud. For example, "It took 12 seconds. Not bad, but let's see if we can get these papers passed out in 11 seconds next time."
- Tell students to begin returning papers back in to the original stack, and then report the time upon completion.
    - Excellent job. Now, pass them back out in 10 seconds. Excellent. Now, pass them back in 8 seconds.
- Pose the following questions to the students as a whole group, one question at a time.
    - How will we measure our rate of passing out papers?
    - *Usingastopwatchorsimilartooltomeasurethenumberofsecondstakentopassoutpapers.*
    - What quantities will we use to describe our rate?
    - *The number of papers passed out and the time that it took to pass them out.* Complete the second and third columns (number of papers and time) on the table as a class.
- Guide the students to complete the ratio column in the table as shown

Example 1: Passing Paper Routine

Trial	Number of papers passed	Time (In seconds)	Ratio of number of papers passed to time	Rate	Unit Rate
1	20				
2	20				

3	20				
4	20				

- Guide questions for the activity
    - From the table, compare the ratio in our trial 1 to the ratio in our trial 4. Are these two ratios equivalent?
    - In my other classes, we pass 18 papers in 12 seconds, and then 18 papers in 9 seconds. A third class passed 20 papers in 16 seconds. How do these compare to our class?
    - How do we easily compare the ratio from my other classes?
- Example 2: Our class by Gender

	Number of Boys	Number of girls	Ratio of Boys to Girls
Class 1			
Class 2			
Whole grade level			

- Are the ratios of boys to girls in the two classes equivalent?
- What could these ratios tell us?
- What does the ratio of the number of boys to the number of girls in class 1 to the ratio of the number of boys to the number of girls in the entire grade level?
- Stress on the concept on equivalent ratios and unit rates.

Exercise 1: Which is the Better Buy?

Family mart is advertising a Back-to-School sale on pencils. A pack of 30 sells for $7.97, whereas a 12-pack of the same brand costs $4.77. Which is the better but? How do you know?

- You may present the solution through a table

	Pack of 30	**Pack of 12**
Relationship		
Ratio		
Rate		
Unit rate		
Unit of Measure		

Session 2: An experience in Relationships as Measuring Rate

- Watch the video clip of Tillman the English bulldog, the Guinness World Record holder for Fastest Dog on a Skateboard.
- At the conclusion of the video, your classmate takes out his or her calculator and says, "Wow that was amazing! That means the dog went about 5 meters in 1 second!" Is your classmate correct, and how do you know?
- After seeing this video, another dog owner trained his dog, Lightning, to try to break Tillman's skateboarding record. Lightning's fastest recorded time was on a 75-meter stretch where it took him 15.5 seconds. Based on these data, did Lightning break Tillman's record for fastest dog on a skateboard? Explain how you know.

Practice Exercise

1. Find each rate and unit rate.
    a. 420 miles in 7 hours
    b. 360 customers in 30 days
    c. 40 meters in 16 seconds
    d. 7.96 for 5 pounds
2. Write three ratios that are equivalent to one given: The ratio of right-handed to left-handed students is 18:4.
3. Mr. Sam has 16 homework papers and 14 mini-tests to return. Ms. My has 64 homework papers and 60 mini-test to return. For each teacher, write a ratio to represent the number of homework papers to number of mini-tests they have to return. Are the ratios equivalent? Explain.
4. Jonathan's parents told him that for every hours of homework or reading he completes, he would be able to play hours of video games. His friend Lucas's parents told their son that he could play 30 minutes for every hour of homework or reading time he completes. If both boys spend the same amount of time on homework and reading this week, which boy gets more time playing video games?

Session 3: Proportional Relationships

Activity 1: Pay by the Ounce Frozen Yogurt

A new self-serve frozen yogurt store opened this summer that sells its yogurt at a price based upon the total weight of the yogurt and its toppings in a dish. Each member of Isabelle's family weighed his dish, and this is what they found. Determine if the cost is proportional to the weight.

Weight (ounces)	12.5	10	5	8
Cost ($)	5	4	2	3.2

- Discuss the following questions:

    1. Does everyone pay the same cost per ounce? How do you know?

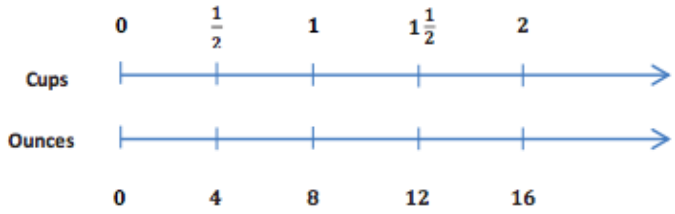
    2. Isabelle's brother takes an extra-long time to create his dish. When he puts it on the scale, it weighs 15 ounces. If everyone pays the same rate in this store, how much will his dish cost? How did you calculate this cost?

    3. What happens if you don't serve yourself any yogurt or toppings, how much do you pay?

- At this point, generate answers from the students to a point where they can make relationship between quantities.

Activity 2: A Cooking Cheat Sheet

At the back of a recipe book, a diagram provides easy conversions to use while cooking.



- Discuss the following questions:

    1. What does the diagram tell us?

    2. Is the number of ounces proportional to the number of cups?

    3. How many ounces are there in 4 cups? 5 cups? 8 cups? How do you know?

Practice Exercise

- **Summer Job**

    Alex spent the summer helping out at his family's business. He was hoping to earn enough money to buy a new $220 gaming system by the end of the summer. Halfway through the summer, after working for 4 weeks, he had earned $112. Alex wonders, "If I continue to work and earn money at this rate, will I have enough money to buy the gaming system by the end of the summer?"

    To determine if he will earn enough money, he decided to make a table. He entered his total money earned at the end of Week 1 and his total money earned at the end of Week 4.

Week	**0**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**

Total earnings		**$28**			**$112**				

- Group discussions

  1. Discuss in your group to answer Alex's question.
  2. Are Alex's total earnings proportional to the number of weeks he worked? How do you know?

- How do we know if two quantities are proportional to each other?
- How can we recognize a proportional relationship when looking at a table or a set of ratios?

Session 4: Proportional and Non-Proportional Relationships in Tables

- Present the situation below

  You have been hired by your neighbors to babysit their children on Friday night. You are paid $8 per hour. Complete the table relating your pay to the number of hours you worked.

Hours Worked	Pay (in dollars)
1	
2	
3	
4	
4 ½	
5	
6	
6.5	

  Based on the table, is the pay proportional to the hours worked? How do you know?

  - Explain how you completed the able.
  - How did you determine the pay for 4 ½ hours?

- How could you use the information to determine the pay for a week in which you worked for 20 hours?
- How many ways can the answer be determined?

- Give more practice exercise for this activit**y.**

Session 5: Proportional and Non-Proportional Relationships in Graphs
Opening activity

Jola sold candy bars to help raise for her scouting troop. The table show the amount of candy she sold compared to the money she received.

x Candy bars sold	y Money received ($)
2	3
4	5
8	9
12	12

Is the amount of candy bars sold proportional to the money Jola received? How do you know?

Exploratory Challenge: From a Table to a Graph

Make a simple recall on the following: coordinate plane, x-axis, y-axis, origin, quadrants, plotting points, and ordered pairs.

- Have the students plot the points in the coordinate grid.
- Give 2 or 3 more examples on from a table to a graph activity.

The Asian International School
Unit Backward Design
Mathematics, Elementary, 2018-2019
Chapter 1, Unit 2: Unit Rate and Constant of Proportionality

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships
- Represent proportional relationships by equations. *For example, if total cost is proportional to the number of items purchased at a constant price, the relationship between the total cost and the number of items can be expressed as t = pm*
- explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where is the unit rate.

Understanding(s):

Students will understand that…

- a ratio is an ordered pair of numbers which are not both zero.
- a ratio is denoted by A : B to indicate the order of the numbers
- two ratios A:B and C:D are equivalent ratios if there is a nonzero number c such that C = cA and D = cB
- the numerical part of the rate is called the **unit rate** and is simply the value of the ratio

Essential Question(s):

- What is the other name for the constant that relates the measure of two quantities?
- What is the importance of finding the unit rate?

Knowledge:

Students will know …

- unit rate is the constant of proportionality
- proportional relation can be written as y = kx, where k is the constant of proportionality
- proportional relationship can be observe in graphs or in tables

Skills:

Student will be able to:

- identify the constant of proportionality in tables, graphs, equations and diagrams
- represent a proportional relationship in equations
- Explain what point (x,y) on the graph of proportional relations mean in terms of the given situation

Stage 2 - Assessment Evidence

Performance Task(s):	**Other Evidence:**
• Make a list of a scenario in your real life that you used proportional relations. • Having learned about proportional relations, is there a scenario that may have different end when you applied your knowledge on proportional relations?	• Homework • Mini-Tests

Stage 3 – Learning Plan

Learning Activities:

Session 1: Unit Rate as the Constant of Proportionality

- Opening Problem:

  Wildlife conservationists are concerned that the deer population might not be constant across the National Forest. The scientists found that there were 144 deer in a 16 square mile area of the forest. In another part of the forest, conservationists counted 117 deer in a 13 square mile area. Yet a third conservationist counted 24 deer in a 216 square mile plot of the forest. Do conservationists need to be worried?

  Ref: https://www.kyrene.org/cms/lib/AZ01001083/Centricity/Domain/971/7NY-%20Lesson%207b-%20Determining%20the%20Constant%20of%20Proportional%20Student.notebook.pdf

- Give time for students to discuss their answers to the opening problem. Share after their group discussions
- Give "You Need What?" Activity

Session 2: Unit Rate as the Constant of Proportionality continued

Susan and John are buying cold drinks for a neighborhood picnic. Each person is expected to drink one can of soda. Susan says that if you multiply the unit price for a can of soda by the number of people attending the picnic, you will be able to determine the total cost of the soda. John says that if you divide the cost of a 12-pack of soda by the number of sodas, you will determine the total cost of the sodas. Who is right and why?

- Practice Exercises

  Answer the following:

  1. Bananas are $0.59/pound.

a. What is the constant of proportionality or k?

b. How much will 25 pounds of banana cost?

2. The dry cleaning fee for 3 pants is $18.

   a. What is the constant of proportionality?

   b. How much will the dry cleaner charge for 11 pairs of pants?

3. For every $5 that Nicah saves, her parents give her $10.

   a. What is the constant of proportionality?

   b. If Micah saves $150, how much money will her parents give her?

Session 3-4: Representing Proportional Relations with Equations

Opening Problem: Do we have enough gas to make it to the next station?

Your mother has accelerated onto the interstate beginning a long road trip and you notice that the low fuel light is on, indicating that there is a half a gallon left in the gas tank. The nearest gas station is 26 miles away. Your mother keeps a log where she records the mileage and the number of gallons purchased each time she fills up the tank. Use the information in the table below to determine whether you will make it to the gas station before the gas runs out. You know that if you can determine the amount of gas that her car consumes in a particular number of miles, then you can determine whether or not you can make it to the next gas station.

Gallons	Miles Driven
8	224
10	280
4	112

a. Find the constant of proportionality and explain what it represents in this situation.

b. Write equation(s) that will relate the miles given to number of gallons of gas.

c. Knowing that there is a half-gallon left in the gas tank when the light turns on, will she make it to the nearest gas station?

d. Using the equation found in part (b), determine how far your car can travel on 18 gallons of gas. Solve the problem in two ways: once using the constant of proportionality and once using equation.

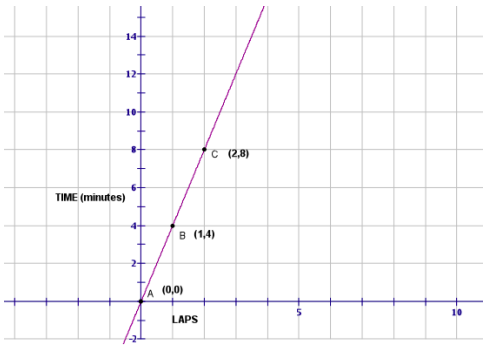
More references: https://www.onlinemathlearning.com/proportional-relationships-equations.html

https://www.slideserve.com/iria/representing-proportional-relationships-with-equations

Session 5 - 6: Interpreting Graphs of Proportional Relationships

- **Student Probe**

  Susan runs three laps at the track in 12 minutes. A graph of this proportional relationship is shown below.



  Explain the meaning of A(0,0); B(1,4) and C(2,8).

- Follow the activity on http://mathinterventions.org/files/uploads/Graphs_of_Proportional_Relationships.pdf
- Example 1:

  Grandma's Special Chocolate Chip Cookie recipe, which yields □ dozen cookies, calls for □ cups of flour. Using this information, complete the table below.

  a. Create a table comparing the amount of flour used to the amount of cookies.

Number of Cups of Flour	Number of dozens of cookies
3	4
6	
12	

  b. Is the number of cookies proportional to the amount of flour used? Explain why or why not.

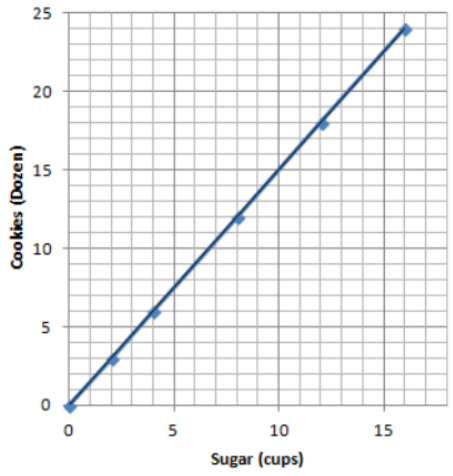
  c. Model the relationship on a graph.

d. Does the graph show the two quantities being proportional to each other? Explain.

e. Write an equation that can be used to represent the relationship.

Reference:
https://www.mathinmind.com/uploads/6/1/3/0/61303639/cookies_proportional_relationship_task.pdf

- Example 2: Below is a graph modeling the amount of sugar required to make Grandma's special chocolate chip cookies.



a. Record the coordinates from the graph. What do these ordered pairs represent?

b. Grandma has 1 remaining cup of sugar. How many dozen cookies will she be able to make? Plot the point on the graph.

c. How many dozen of cookies can Grandma make if she has no sugar? Can you graph this on the coordinate grid? What do we call this point?

- Give 2 or 3 practice exercises (https://www.mathworksheetsland.com/8/7graphprop/ip.pdf)

The Asian International School
Unit Backward Design
Mathematics, Elementary, 2018-2019
Chapter 1, Unit 3: Ratio and Rates Involving Fractions

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships
- Represent proportional relationships by equations. *For example, if total cost is proportional to the number of items purchased at a constant price, the relationship between the total cost and the number of items can be expressed as t = pm*
- explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points (0, 0) and (1, r) where is the unit rate.

Understanding(s):

Students will understand that…

- a ratio is an ordered pair of numbers which are not both zero.
- a ratio is denoted by A : B to indicate the order of the numbers
- two ratios A:B and C:D are equivalent ratios if there is a nonzero number c such that C = cA and D = cB
- the numerical part of the rate is called the **unit rate** and is simply the value of the ratio

Essential Question(s):

- What is the other name for the constant that relates the measure of two quantities?
- What is the importance of finding the unit rate?

Knowledge:

Students will know …

- unit rate is the constant of proportionality
- proportional relation can be written as y = kx, where k is the constant of proportionality
- proportional relationship can be observe in graphs or in tables

Skills:

Student will be able to:

- identify the constant of proportionality in tables, graphs, equations and diagrams
- represent a proportional relationship in equations
- Explain what point (x,y) on the graph of proportional relations mean in terms of the given situation

Stage 2 - Assessment Evidence

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Unit Backward Design
Mathematics, Elementary, 2018-2019

Performance Task(s):	**Other Evidence:**
• Make a list of a scenario in your real life that you used proportional relations. • Having learned about proportional relations, is there a scenario that may have different end when you applied your knowledge on proportional relations?	• Homework • Mini-Tests

Stage 3 – Learning Plan

Learning Activities:

Session 1: Unit Rate as the Constant of Proportionality

Opening Activity

During last workout, Liza ran $2\frac{1}{4}$ miles in 15 minutes, and her friend Jane ran $3\frac{3}{4}$ miles in 25 minutes. Each girl thought she was the faster runner. Which girl is correct?

Think-Pair-Share

T : (Think) Student will think about the problem and try to solve in on their own.

P : (Pair) Each student will choose a partner or group

S : (Share) Students share their thinking with their partner or group.

- Teacher will expand the "share" into a whole class discussions.
- Allow class discussions for this activity. Call some students to show a possible solution to this problem. From here you can figure out how students calculate expressions involving fractions.
- Encourage students to organize their solution in a form a table.

Liza

Time (minutes)	Time (hours)	Distance (miles)
15		

Jane

Time (minutes)	Time (hours)	Distance (miles)
25		

30		
45		
60		
75		

50		
75		
100		

- This is students' first experience evaluating complex fractions. Be sure to relate the process of evaluating complex fractions to division of fractions. Solutions can be presented in different ways so accept all valid solutions.

Activity 2

A turtle walks $\frac{7}{8}$ of a mile in 50 minutes. What is the unit rate when the turtle's speed is expressed in miles per hour?

a. To find the he turtle's unit rate, Minh wrote the following ratio. Explain how fraction $\frac{5}{6}$ was obtained.

$$\frac{\left(\frac{7}{8}\right)}{\left(\frac{5}{6}\right)} =$$

b. Determine the unit rate when the turtle's speed is expressed in miles per hour.

Activity 3

Sally is making a painting for which she is mixing red paint and blue paint. The table shows the different mixtures being used.

Red paints (in Liters)	Blue paints (in Liters)
$1\frac{1}{2}$	$2\frac{1}{2}$
$2\frac{2}{5}$	4
$3\frac{3}{4}$	$6\frac{1}{4}$
4	$6\frac{2}{3}$

a. What is the unit rate for the values of the amount of blue paint to red paint?

1.2	2
1.8	3

b. Is the amount of blue paint proportional to the amount of red paint?

c. Describe in words what the unit rate means in the context of this problem.

Activity 4

Which is the better buy? Show your work and explain your reasoning.

1. a. $3\frac{1}{3}$ lb of turkey for $8.5
   b. $2\frac{1}{2}$ lb of turkey for $4.25

2. a. $\frac{3}{4}$ lb bag of nuts for $5.95
   b. $\frac{1}{2}$ lb bag of nuts for $3.89

Practice Exercises

1. Ethan writes 1/6 of a page in 1/12 of a minute. How much time does it take him to write a full page?
2. William fills 1/3 of a water bottle in 1/6 of a minute. How much time will it take him to fill the bottle?
3. Michael plays 1/5 of a song in 1/15 of a minute. How much time will it take him to play an entire song?
4. Gabriel used 1/3 of a liter of milk to make 1/9 of a jug of tea. How much milk is required to fill the jug?
5. Isaac used 1/4 of an ounce of nuts to make 1/12 pound of cake. How many ounces of nuts are needed to make a cake?

Activity 5: Bargains

A retail clothing store advertises the following sale: Shirts are off the original price; pants are off the original price, and shoes are off the original price (called the discount rate).

a. If a pair of shoes cost $40 and is advertised at $\frac{1}{4}$ off the original price, what is the sales price?

b. At Peter's Pants Palace a pair of pants that usually sell for $33.00. If Peter advertises that the store is having $\frac{1}{4}$ off sale, what is the sale price of Peter's pants?

The Asian International School
Unit Backward Design
Mathematics, Elementary, 2018-2019
Chapter 1, Unit 4: Ratios of Scale Drawing

<table>
<tr><th colspan="2">Stage 1 - Desired Results</th></tr>
<tr><td colspan="2">Established Goal(s):

At the end of the unit, students will be able to:

• Use unit rate as the scale factor of a drawing / figure

• Draw an enlargement or reduction of any given figures</td></tr>
<tr><td>Understanding(s):

Students will understand that…

• enlargement and reduction of sizes requires knowledge on proportions

• multiplying a constant to the unit rate will either reduce or enlarge a figure</td><td>Essential Question(s):

• How are maps measurements as to its actual measurement?

• How do we use ratio in our real life?</td></tr>
<tr><td>Knowledge:

Students will know …

• enlargement and reduction of sizes requires knowledge on proportions

• multiplying a constant to the unit rate will either reduce or enlarge a figure</td><td>Skills:

Student will be able to:

• Understand that scale drawing is either the reduction or enlargement of a two-dimensional picture.
• Compare scale drawing picture with the original picture</td></tr>
<tr><th colspan="2">Stage 2 - Assessment Evidence</th></tr>
<tr><td>Performance Task(s):

• Make a route direction from your house to school. Bea able to determine the scale factor between your map and the actual distance in your map.</td><td>Other Evidence:

• Homework

• Mini-Tests</td></tr>
<tr><th colspan="2">Stage 3 – Learning Plan</th></tr>
<tr><td colspan="2">Learning Activities:</td></tr>
</table>

Session 1: Unit Rate as the Constant of Proportionality

Opening Activity

Can you guess the image?

- For each scale drawing, have students identify if it is a reduction or an enlargement of the actual object in real life or of the given original picture.
- What are possible uses for enlarged drawings/pictures?
- What are the possible purposes of reduced drawings/pictures

Key Ideas

Scale Drawing: A drawing in which all lengths between points of figures in the drawing are reduced on enlarged proportional to the lengths in the actual picture. A constant of proportionality exist between corresponding lengths of the two images.

Reduction: The lengths in the scale drawing are smaller than those in the actual drawing or picture.

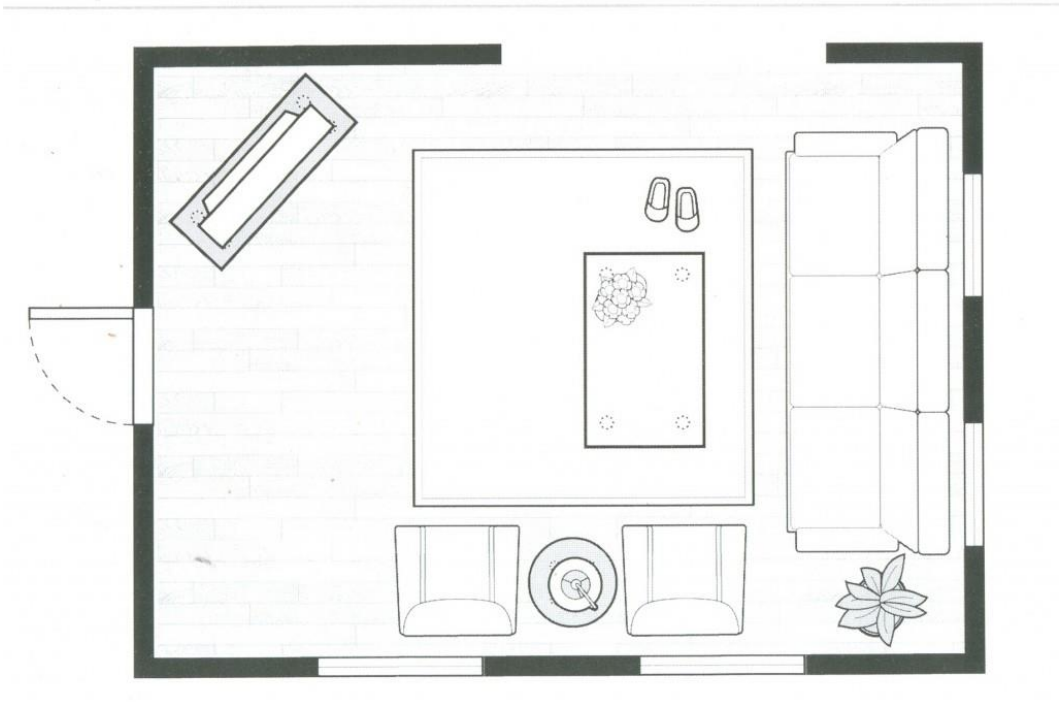
Enlargement/Magnification: The lengths in the scale drawing are larger than those in the actual drawing or picture.

One-to-one Correspondence: Each point in one figure corresponds to one and only one point in the second figure.

- Give enough practice and activity for your class about reduction and enlargement of images or pictures.

Plan of a living room

Scale 1:50



a. What are the actual measurements of the room?

b. What is the area of the room?

c. The sofa is 3m long, how long is it on the drawing?

d. What would be the actual size of the rug in the middle of the room (do not round up)?

e. There is a TV in the corner of the room what is the actual length of the real TV?

f. There is a round table on the drawing. This has a diameter of 1.2cms. Using the formulae πr^2, where $\pi = 3.14$ What would the area of the actual table be?

Model worksheet

Scale 1:38

You have been given a model car. This is a replica of a Ferrari 250 GTO.

Using your model find the actual size of the car (length, height and width)

How long is the bonnet of the model car? what is the measurement of the real bonnet?

Find the actual measurements of :-

Door

Windscreen

Front grill

Back number plate.

The wheels on the model have a diameter of 1.8cms. Find the circumference of the actual wheel using the formulae πd, where $\pi = 3.14$

Model worksheet

Scale 1:80

You have been given a model truck. This is a replica of an American Peterbilt truck. Using your model find the actual size of the truck (length, height and width)

Find the area of the front grill on the model and on the actual truck.

This truck has a trailer. The actual trailer is 480 cms, what size would the scale model be?

The wheels on the model have a diameter of 1.1cms. Find the circumference of the actual wheel using the formulae πd, where π = 3.14

Find the actual measurements of:-

Door

One window

Bonnet

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Chapter 2, Unit 1: Solve Problems Involving Equations

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- Generate equivalent expressions using associative and commutative property
- Write expressions in expanded and standard form

Understanding(s):

Students will understand that…

- Letters or variables are used to represent a number in an expression
- Equivalent expressions can be made using algebraic operations

Essential Question(s):

- How are maps measurements as to its actual measurement?
- How do we use ratio in our real life?

Knowledge:

Students will know …

- different ways of writing equivalent expressions

Skills:

Student will be able to:

- Generate equivalent expressions using associative and commutative property
- Write expressions in expanded and standard form

Stage 2 - Assessment Evidence

Performance Task(s):

- Make a route direction from your house to school. Bea able to determine the scale factor between your map and the actual distance in your map.

Other Evidence:

- Homework
- Mini-Tests

Stage 3 – Learning Plan

Learning Activities:
Opening Activity

Each envelop contains a number of triangles and a number of squares. For this exercise, let ***t*** represents the number of triangles and let ***s*** represents the number of squares.

- ***(If the materials are not available, you can modify this activity that follows the same exploratory learning approach or you can limit the number of envelops good for 4 or 5 groups)***

a. Write an expression using t and s that represents the total number of sides in your envelop.
   - Guide students in writing the correct expression for this activity

b. You and your partner have the same number of triangles and square in your envelops. Write an expression that represents the total number of sides that you and your partner have.
   - Have some students write their answer on the board. If possible, write more than one expression to represent this total.

c. Each envelop in the class contains the same number of triangles and squared. Write an expression that relates the total number of sides in the room.
   - Answer depends on the number of students in the class

- Discuss any variations of answers in the class. Have the student understand that these variations are equivalent. Introduce the concept of equivalent expressions.
- Choose one student from the class and count the number of triangles and squares. Record the values of ***t*** and ***s***.

d. Use the given values of t and s in your expression in part (a) to determine the number of sides that should be found in your envelop.
   - You can ask one student to write the answer on the board.

e. Use the same values for t and s and your expressions from part (b) to determine the number of sides that should be contained in your envelop and your partner's envelop combined.
   - Present the solution in different variations

f. Use the same values for t and s and your expressions in part (c) to determine the number of sides that should be contained in all envelops.
   - Answer will depend on the number of envelops in the class.

- You may also want to open their own envelop and use the information to answer part (a) to part (c).

Key Ideas

Algebraic Expressions – a mathematical phrase that contains a combination of numbers, variables, and operational symbols (+ , – , × , ÷)

Variable – letter that can represent one number or more numbers

Commutative Property of Addition – changing the order of the addends does not change the sum

Grouping property with addition/subtraction

Example 1

Rewrite 5x + 2x and 5x – 2x in expanded form then in standard form.

$$\overbrace{\underbrace{x + x + x + x + x}_{5x} + \underbrace{x + x}_{2x}}^{7x} = 7x$$

$$5x - 2x = \overbrace{x + x + x + x + x}^{5x} - \underbrace{x + x}_{2x} = 3x$$

OR

5x + 2x = (5 + 2) x = 7x 5x – 2x = (5 – 2) x = 3x

Example 2

Find the sum of 3x + 1 and 4x.

- Possible solution

  (3x + 1) + 4x given

  3x + (1 + 4x) associative property of addition

3x + (4x + 1)	commutative property of addition
(3x + 4x) + 1	associative property of addition
(3 + 4) x + 1	combine like terms
7x + 1	equivalent expression to the given

- Guide students in answering the following for practice.

  Write an equivalent expression for the following using properties of expressions

  a. Find the sum of $-4a + 3$ and $5a - 2$.
  b. Find the product of 4a and 3.
  c. 2(4y)
  d. 2(3a) + 4(2b)
  e. $3(2x) - 4(2y) + 3x + 4y$

More Practice

1. Write an equivalent expression to 8x + 5 + 7x + 4 by combining like terms.
2. Find the sum of $(2a + 3b - 2)$ and (4b + 5).
3. Write the expression in standard form: $5(2a) + 3(-2b) + 3 \cdot c \cdot b$

- Provide more practice for this lesson.
- Introduce additive inverse as opposites

Expression	Additive inverse
1	−1
5	−5
−7	3
$\frac{3}{2}$	−
a	−a

3a	$-3a$
a + 4	$-(a+4)$
$3a - 4$	$-3a + 4$

- Stress that the sum of an expression and its opposite is always equal to zero.
- Introduce subtracting expressions using the idea of additive inverse.

  Subtract: $(20a + 8) - (15a + 2)$

  $(20a + 8) - (15a + 2)$ given

  $= 20a + 8 + (-15a) + (-2)$ additive inverse

  $= 20a + (-15a) + 8 + (-2)$ commutativity

  $= (20 + (-15))\, a + (8 + (-2))$ associativity

  $= 5a + 6$ combining like terms

Combining Expressions Vertically

Example 3

a. Find the sum by aligning the expressions vertically

   $(3a + 5b + 6) + (5a + 2b + 3)$

b. Find the difference by aligning the expressions vertically

   $(2a + 7b - 3) - (a - 3b + 1)$

Simplifying Algebraic Expressions

To simplify algebraic expressions means to rewrite it in such a way that it is easier to read and understand.

3 Steps

- Combine like terms
- Perform basic operations (if possible)
- Simplify exponents

Practice Exercise

A. Simplify the following expressions.

1. $4a + 2 - 3a - 5$
2. $5 + 3(4b - 3)$
3. $2(-3c + 1) - 4$
4. $7 - (5d - 8)$
5. $12e + 3(4 - 5e)$
6. $8m - 3 + 4m + 5$
7. $7 + 2(3 - 5m)$
8. $-3(-2 - m) - 5$
9. $8m - (2m - 6m)$
10. $(13m - 2) - (2 - 13m)$

B. Simplify the following expressions.

1. $b - 4 + 3 - 5b$
2. $18 + 2(3n + 5)$
3. $4(-4x + 4) - 4$
4. $14 - (4h + 3)$
5. $21t + 5(2 - 5t)$
6. $18d + 13 - 15 + 3d$
7. $9 + 4(4 - 3d)$
8. $19d - (8d + 12) - 14$
9. $14d - (16d - 8d)$
10. $(27d + 5) - (14 - 5d)$

Evaluating Algebraic Expressions

A. Evaluate each expression using the indicated value.

1. $3 - 2m - 5m$ use $m = 4$
2. $3a + 4 - 5a$ use $a = 5$
3. $-4(-2 + 3k)$ use $k = 3$
4. $3(e + 1) - 4e$ use $e = 2$
5. $-3(2r - 5) + 8$ use $r = 7$
6. $5 + 7b - 3b$ use $b = 8$
7. $5r + 8 - 6r$ use $r = 4$
8. $-2(-3 + 4e)$ use $e = 9$
9. $5(2a + 3) - 5a$ use $a = 7$
10. $-6(4d + 3) + 4$ use $d = 3$

B. Evaluate each expression using the indicated value.

1.	$15 - 8c + 3c$	use $c = 4$	6.	$8 + 12m - 4m$	use $m = 6$
2.	$8b + 3 - 3b$	use $b = 6$	7.	$9a + 12 - 18a$	use $a = 10$
3.	$-6(-2 + 4m)$	use $m = -2$	8.	$6(-4 + 7g)$	use $g = 8$
4.	$5(n + 5) - 5n$	use $n = 2a$	9.	$8(7f + 1) - 2f$	use $f = 2a$
5.	$-7(4p - 5) + 9$	use $p = a + 1$	10.	$-4(2k + 8) + 7$	use $k = a + 1$

The Asian International School
Unit Backward Design
Mathematics, Elementary, 2018-2019
Chapter 2, Unit 2: Solve Problems Involving Equations

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- write mathematical sentence in symbols
- solve problems involving equations

Understanding(s):

Students will understand that…

- Letters or variables are used to represent a number in an expression
- Equivalent expressions can be made using algebraic operations

Essential Question(s):

- How does knowledge in algebra help a person in solving life-related problems?
- Why do we study how to interpret real life situations in mathematical figures?

Knowledge:

Students will know …

- words associated with the fundamental operations
- mathematical sentences can be written in symbols

Skills:

Student will be able to:

- Represent addition, subtraction, multiplication and division by using the symbols of algebra
- Solve problems involving algebraic equations and inequalities

Stage 2 - Assessment Evidence

Performance Task(s):

- Create a mathematical brochure in using knowledge in algebra in solving real-life problems.

Other Evidence:

- Homework
- Mini-Tests

Stage 3 – Learning Plan

Learning Activities:
Word associated with operations

Addition : increased by, plus, more than, sum, added to, . . .

Subtraction : fewer than, less than, difference between, decreased by, . . .

Multiplication : the product of, times, multiplied to, . . .

Division : quotient of, divided by, divided among, shared between**, …**

Write each phrase using symbols.

1. the sum of a and b
2. d plus e
3. m increased by 5
4. 7 more than p
5. 5 minus b
6. 8 times k
7. the product of 6 and t
8. the product of 10, m, and n
9. twice the sum of x and y
10. the quantity a plus b increased by the quantity a minus b

Write each phrase using symbols.

1. 10 more than y
2. h decreased by 4
3. the product of 3 and the sum of n plus m
4. 8 less than r
5. the product of a plus b and x plus y
6. the sum of 3 times m and n
7. the sum of twice x and y
8. the quotient of a pus b and c
9. the product of m and 7 more than n
10. twice the difference of u and v

Key Ideas

An **equation** is a mathematic a statement that two expressions are equal.

A **solution** for an equation is any value for the variable that makes the equation a true statement.

Problem Solving

1. The ages of three sisters are consecutive integers. The sum of their ages is 48. Calculate their ages.
2. Sophie pays $18 membership fee for an online music store.
   a. If she also buys two songs from a new album at a price of $0.99 each, what is the total cos?

b. If Sophie purchases **n** songs for $0.99 each, write an expression for the total cost.
c. Sophia's friend has saved $118 but is not sure how many songs she can afford if she buys the membership and some songs. Use the expression in part (b) to write an equation that can be used to determine how many songs Sophie's friend can buy.

3. For a walk-a-thon a sponsor committed to give you a flat fee of $5 plus $2 for every mile **m** you walk.
   a. Write an expression for the total amount you will collect from your sponsor at the end of the walk-a-thon.
   b. How much did you raise if you walked for 20 miles?

4. A cell phone company charges $40 per month plus a $35 activation fee.
   a. Write an expression for the total cost of m months.
   b. How much is paid for 10 months of service?

Solving Inequalities

Solving inequalities is very similar to solving equations.

Practice Exercise

A. Solve the following equations.

1. $a + 3 < 7$
2. $5b - 3 > 18$
3. $3c + 5 > 2c - 3$
4. $8 + 5d < 9d + 1$
5. $12 + 5e \geq 2e - 15$
6. $4g - 5 > 5g$
7. $9h - 15 < 3$
8. $4k + 8 > 7k - 8$
9. $18 + 3f < 5f + 4$
10. $9 - 3j \leq 9 - 3j$

B. Find the solution in the following equations.

1. $2m + 5 < 9$
2. $3(2n - 3) > 18$
3. $(3p + 5) + (4p - 1) > 4p - 3$
4. $14 + 5t < 2(9t + 1)$
5. $5(1 + 4s) \geq 2(s - 1)$
6. $5 - 8m < 2(5 - 8m)$
7. $4(3n - 1) < 20$
8. $(3p + 6) - (3p + 4) < 2p + 7$
9. $18 + 4t \leq 3(6 + t)$
10. $-(2 + 3s) \leq 3(2s + 1)$

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Unit Backward Design
Mathematics, Elementary, 2018-2019
Chapter 2, Unit 3: Solve Problems Involving Geometry

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- write mathematical sentence in symbols involving geometry

Understanding(s):

Students will understand that…

- Letters or variables are used to represent a number in an expression
- Equivalent expressions can be made using algebraic operations

Essential Question(s):

- How does knowledge in algebra help a person in solving life-related problems?
- Why do we study how to interpret real life situations in mathematical figures?

Knowledge:

Students will know …

- words associated with the fundamental operations
- mathematical sentences can be written in symbols

Skills:

Student will be able to:

- write and solve equations related to angles, area and volume

Stage 2 - Assessment Evidence

Performance Task(s):

- Create a mathematical brochure in using knowledge in algebra in solving real-life problems.

Other Evidence:

- Homework
- Mini-Tests

Stage 3 – Learning Plan

Learning Activities:
Key Ideas

Angle Pairs

Linear pairs are angles that form a straight line.

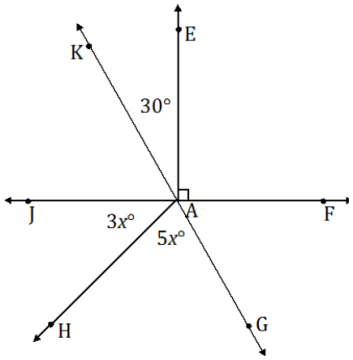
Supplementary pair / supplementary angles are two angles that add up to 180°.

Complementary pair / complementary angles are two angles that add up to 90°.

Opening Activity

A. Write and solve equations for the value of the lettered angle.

1.



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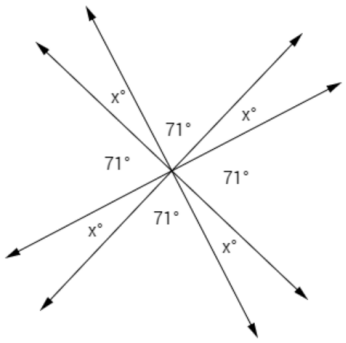

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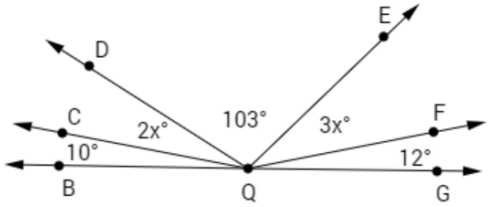

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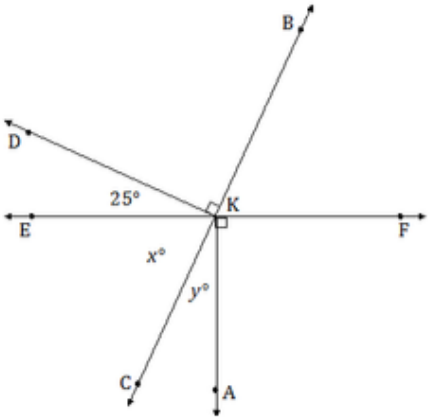

Problem Solving Involving Geometry

A. Use the information to solve the following problems.

1. The longest side of a triangle is six more units than the shortest side. The third side is twice the length of the shortest side. If the perimeter of the triangle is 25 units, write and solve an equation to find the lengths of all three sides of the triangle.

2. The length of a rectangle is (x + 3) inches long, and the width is $3\frac{2}{5}$ inches. If the area is

$15\frac{3}{10}$ square inches, write and solve an equation to find the length of the rectangle.

3. A picture is $8\frac{1}{4}$ feet long is to be centered on a wall that is $16\frac{1}{2}$ feet long. How much space is there from the edge of the wall to the picture?

4. A triangle has a perimeter of 60. If 2 of its sides are equal and the third side is 6 more than the equal sides, what is the length of the third side?

5. The width of a rectangle is 3 feet less than its length. The perimeter of the rectangle is 70 feet. Find its dimensions.

6. In a quadrilateral two angles are equal. The third angle is equal to the sum of the two equal angles. The fourth angle is 30° more than the third angle. Find the measures of the angles in the quadrilateral.

7. The sum of the supplement and the complement of an angle is 130 degrees. Find the measure of the angle.

8. The two angles in a quadrilateral are equal. The third angle is equal to the sum of the two equal angles. The fourth angle is 80° less than the sum of the other three angles. Find the measures of the angles in the quadrilateral.

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Mathematics, Elementary, 2018-2019
Chapter 4, Unit 2: Types of Triangles

<table>
<tr><th colspan="2">Stage 1 - Desired Results</th></tr>
<tr><td colspan="2">Established Goal(s):

At the end of the unit, students will be able to:

• understand interior and exterior angles of a triangle</td></tr>
<tr><td>Understanding(s):

Students will understand that…

• the sum of the angles of a triangle is always equal to 180°</td><td>Essential Question(s):

• Where do we use triangles in our real life?
• What is the importance of having a knowledge on triangles?</td></tr>
<tr><td>Knowledge:

Students will know …

• sum of the angles of a triangle
• measure of the exterior angles of any triangle</td><td>Skills:

Student will be able to:

• calculate the measure of the third angle of a triangle
• determine the measure of the exterior of a triangle</td></tr>
<tr><th colspan="2">Stage 2 - Assessment Evidence</th></tr>
<tr><td>Performance Task(s):

• Make further investigation on the uses of triangles. Share it in class for the performance tasks.</td><td>Other Evidence:

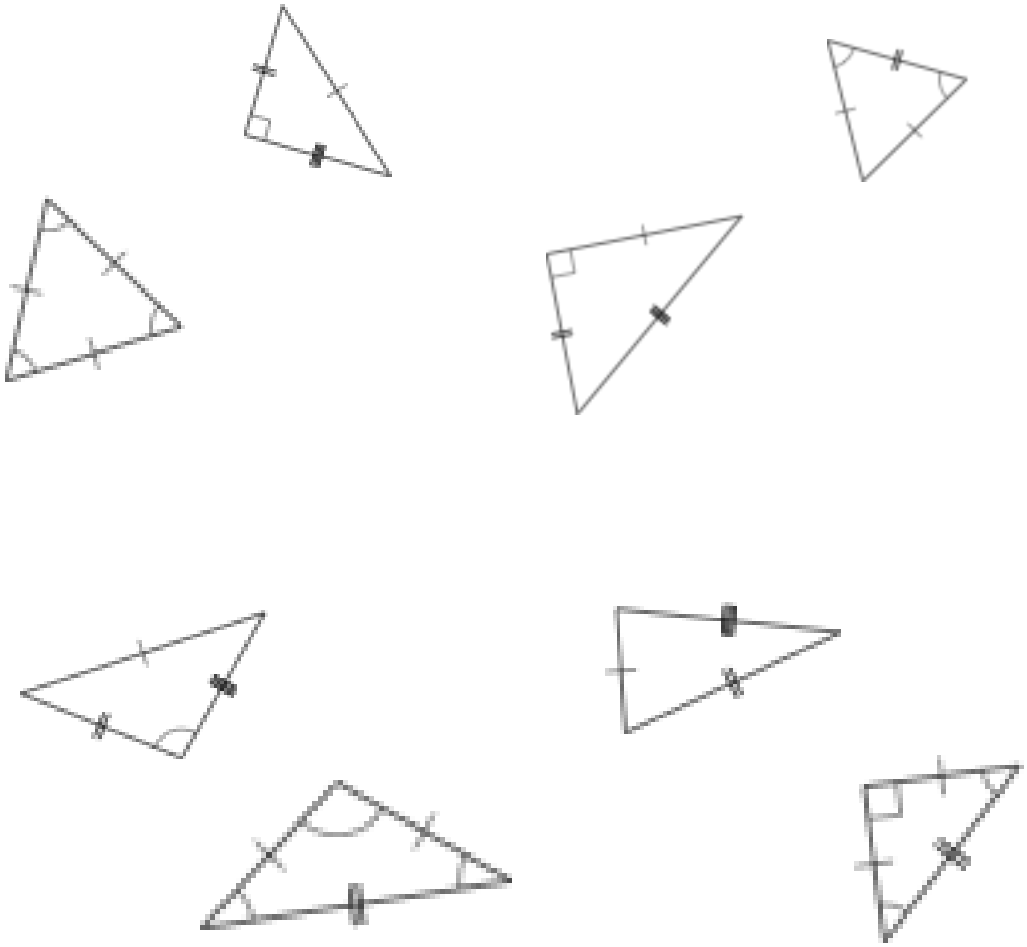
• Homework
• Mini-Tests</td></tr>
<tr><th colspan="2">Stage 3 – Learning Plan</th></tr>
<tr><td colspan="2">Learning Activities:

Triangles according to angles

a. Acute triangle
b. Right triangle
c. Obtuse triangle
Triangle according to sides</td></tr>
</table>

a. Isosceles triangle
b. Equilateral triangle
c. Scalene triangle

Classify each triangle according to sides and angles.



- Make sure the students have a clear understanding on the different types of triangles before giving the next activity.

Draw the following types of triangles.

	Scalene Triangle	**Isosceles Triangle**	**Equilateral Triangle**
Acute Triangle			
Right Triangle			
Obtuse Triangle			

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Chapter 4, Unit 3: Types of Triangles

Stage 1 - Desired Results

Established Goal(s):

At the end of the unit, students will be able to:

- understand interior and exterior angles of a triangle

Understanding(s):

Students will understand that…

- the sum of the angles of a triangle is always equal to 180°
- the sum of any two sides of a triangle is always greater than the third side

Essential Question(s):

- Where do we use triangles in our real life?
- What is the importance of having a knowledge on triangles?

Knowledge:

Students will know …

- median, altitude, and angle bisectors of a triangle
- triangle inequality

Skills:

Student will be able to:

- draw the median, altitude, and angle bisectors of a triangle
- identify the possible sizes of the sides of a triangle

Stage 2 - Assessment Evidence

Performance Task(s):

- Make further investigation on the uses of triangles. Share it in class for the performance tasks.

Other Evidence:

- Homework
- Mini-Tests

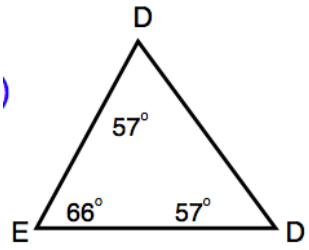
Stage 3 – Learning Plan

Learning Activities:

In a triangle, the sum of any two sides must be greater than the measure of the third side.

A. Order each triangle's sides from largest to smallest

1.



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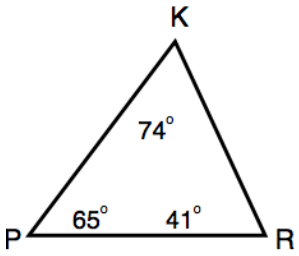

3.

For Δ RXD

m ∠ R = 51 °

m ∠ D = 70°

m ∠ X = 59°

4.

For Δ QBI

m ∠ B = 47°

m ∠ Q = 44 °

m ∠ I = 89°

B. Determine if each set of three numbers can be lengths of the sides of a triangle.

1. 8, 12, 6
2. 7, 9, 2
3. 6, 15, 20
4. 9, 6, 16
5. 8, 10, 7
6. 3, 12, 13
7. 7, 13, 19
8. 5, 7, 16
9. 5, 5, 5
10. 3, 7, 5